

Méthodes topologiques en analyse non linéaire: développements récents -  
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### **Mathematical modeling of household waste recovery**

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The energy crisis caused by the war in Ukraine confirms the urgency to accelerate the development of renewable energies. In addition, the exponential growth in the production of household waste worldwide continues to cause sometimes irreversible environmental damage. The treatment and energy recovery of waste can partly address these issues. However, until now, this field has benefited little from a mathematical modeling approach.

We will present here two dynamic models of waste-to-energy conversion: an economic and a biological one.

The first model describes the problem of a potential investor in the recovery of household waste. We would naturally like to maximize the net profit generated by the sale of the energy produced. We first study the differential system that describes the evolution of a stock of waste, the amount of energy produced, and the capital dedicated to the transformation process in a waste storage and recovery center. Then, we insert decision variables to the system which are both the investment and the part of the waste to be treated. This leads to an optimal control problem that will be solved by the deductive method. A sensitivity analysis with respect to the parameter confirms the robustness of the model. The objective of this investment policy would be to help decision makers to orient themselves towards investing in this activity.

Through a dynamic system of seven equations, the second model describes the biological process of anaerobic digestion of organic matter with acidogenic and methanogenic biomasses. The qualitative analysis shows that an infinite number of non-hyperbolic equilibria induce an attractor that is identified. The results obtained allow to explain the quantities of energy produced according to the initial conditions. Numerical simulations confirm these results and also highlight the impact of the mortality of acidogenic and methanogenic biomasses on biogas production.

**Keywords:** Non-linear dynamical systems - Optimal control - Household waste recovery - waste-to energy - Decision support.

#### **References**

- [1] O.Cherkaoui Dekkaki, N. El Khattabi, N. Raissi, Bioeconomic modeling of household waste recovery. *Math Meth Appl Sci.*, 1{15 (2021).
- [2] I. Ahlamine, A. Alla, N. El Khattabi, Mathematical modelling of the anaerobic digestion process with acidogenic and methanogenic biomasses. Submitted.
- [3] F. Clarke, *Functional Analysis, Calculus of Variations and Optimal Control*. London: Springer-Verlag; (2013).
- [4] J. Harmand, C. Lobry, A. Rapaport, T. Sari, *The Chemostat: Mathematical Theory of Microorganism Cultures*. John Wiley and Sons, Hoboken (2017).
- [5] M. El-Fadel, A. Findikakis, J. Leckie, A numerical model for methane production in managed sanitary landfills. *Waste Management and Research* 7(1), 31-42 (1989).